# Subsurface Wave Measurements

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**Nortek AS**

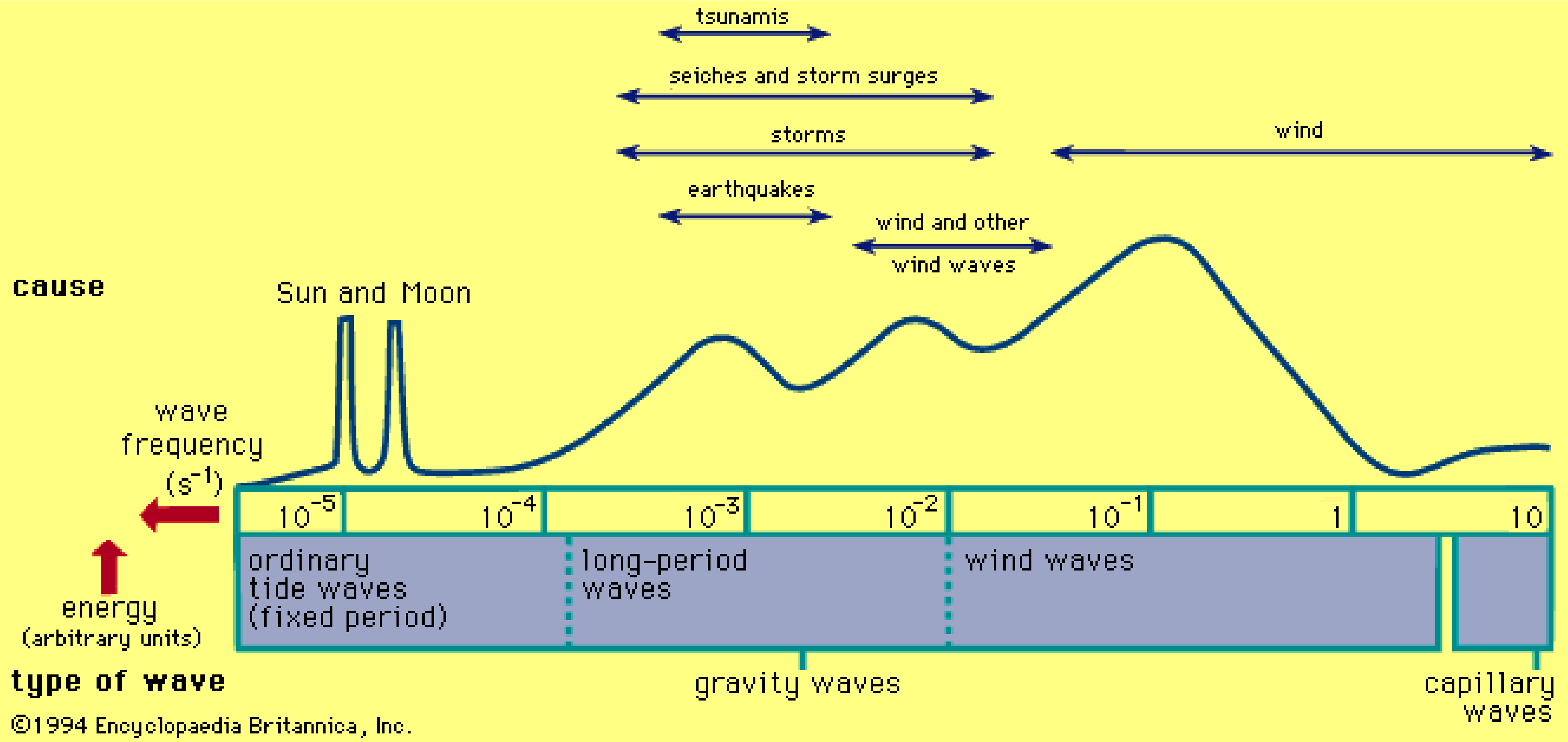
# Overview of Presentation

- Background on bottom mounted wave instruments
- AWAC
- AST
- SUV
- Malaga case study: Breakwater reflection
- Online applications

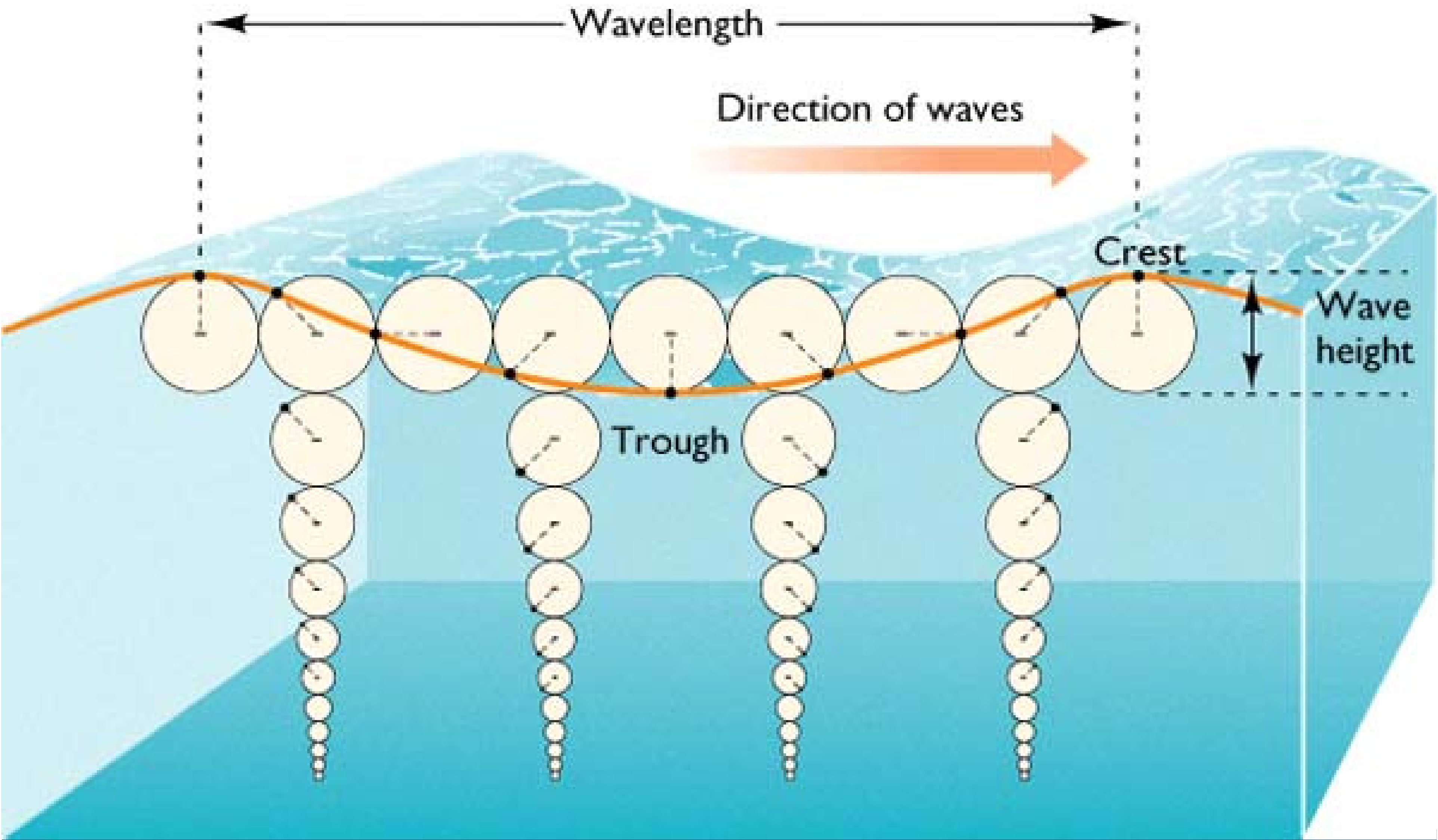
# Assumptions...

- Understanding on Doppler principle
- Doppler profilers
- Basic wave measurement concepts and standard data products such as Hieght, period and directional estimates

# Wave Energy Distribution



# Attenuation of wave properties

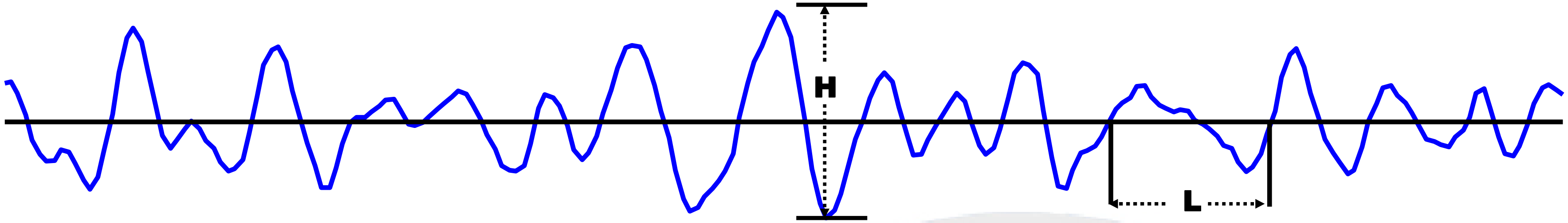


# Orbital Velocity Animation

Link to Orbital Velocity Animation

# Data Products

# Wave Estimates

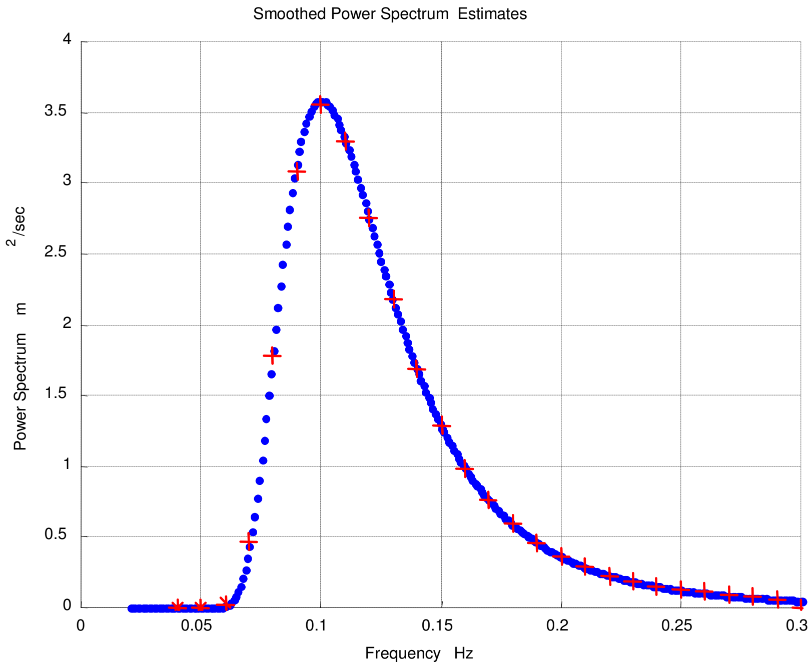



- $H_s$ Significant Wave Height is the "*mean of the 1/3 largest waves in a record*". Requires time series to estimates this directly

- Spectral alternative

$$H_s = 4\sqrt{m0}$$

$$m_k = \int f^k S(f)df$$



# Directional Distribution

$$E(f,\theta) = S(f)D(f,\theta)$$

Two dimensional Energy Distribution

Where D can be considered a probability distribution over Direction

$$\int_0^{2\pi} D(f,\theta)d\theta = 1$$

Criteria Directional Distribution must fulfill

$$D(f,\theta) > 0$$

# Directional Distribution

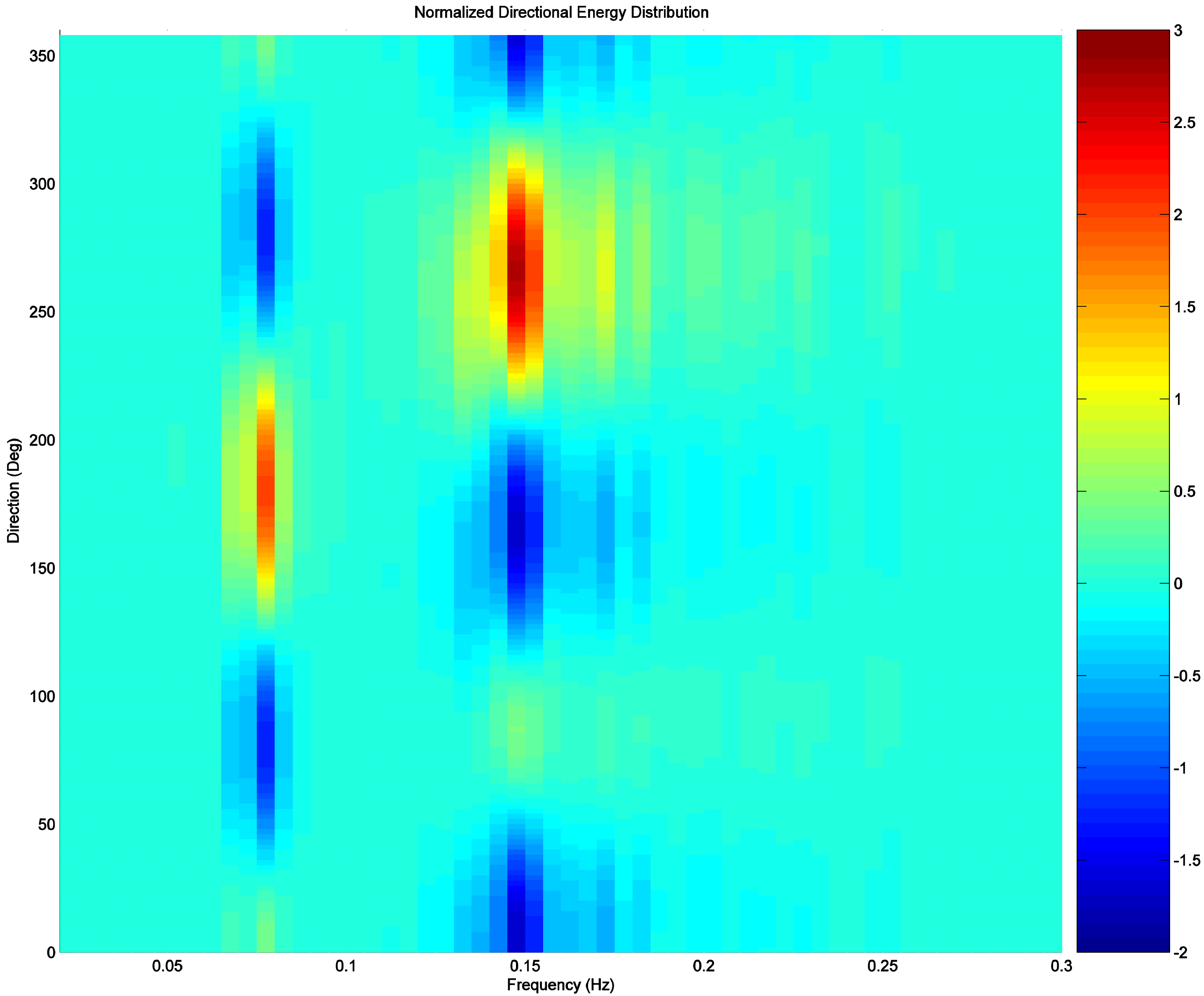
$$D(f,\theta) = \frac{1}{\pi}\left[\frac{1}{2} + \sum_{n}\{a_n \cos n\theta + b_n \sin n\theta\}\right]$$

Longhuet-Higgins 1963

Approximated by first 4 Fourier coefficients derived from various power spectra from directional measures, ie PUV

Problem with negative energy

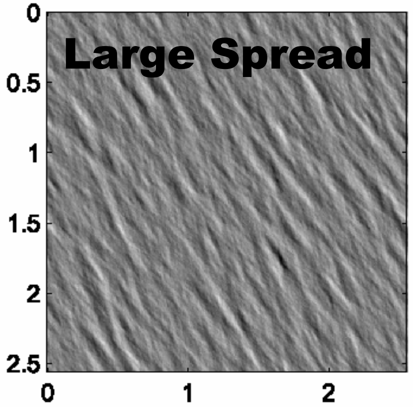
Therefore several methods followed including, Maximum Entropy and Parametric models.

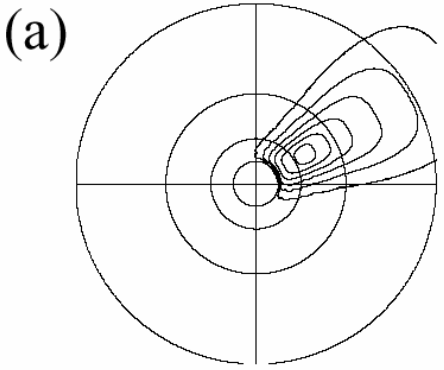


Normalized Directional Energy Distribution
Direction (Deg)
Frequency (Hz)
0
50
100
150
200
250
300
350
0.05
0.1
0.15
0.2
0.25
0.3
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2

# Directional Wave Estimates

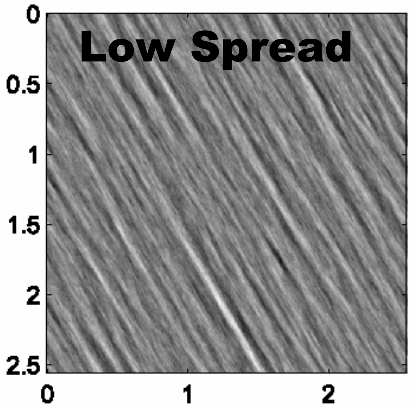
$$D(f,\theta) = \frac{1}{\pi}\left[\frac{1}{2} + \sum_n \{a_n \cos n\theta + b_n \sin n\theta\}\right]$$

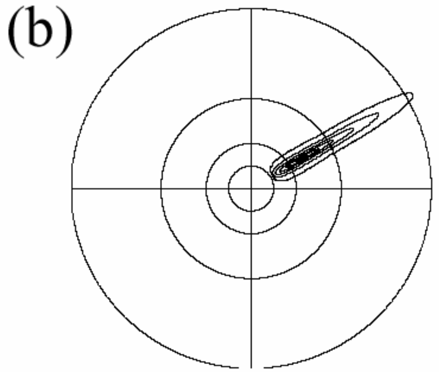



**Mean direction**

$$\theta_1(f) = \arctan(b_1(f)/a_1(f))$$




**Spreading (directional variance)**

$$\sigma = \sqrt{2(1-r_1)}$$

$$r_1 = \sqrt{a_1^2 + b_1^2}$$

# A Brief History of Bottom Mounted Wave Measurements

NORTEK AS

## PUV (1970's)

**Dramatic attenuation: limited long waves and Coastal waters.**

## Doppler Profiler (mid 1980's)

**Near surface current measurements:**
**Roughly doubles deployment depth but still depth limited**
**By projected array size.**

## Directional measurements with AST (2002)

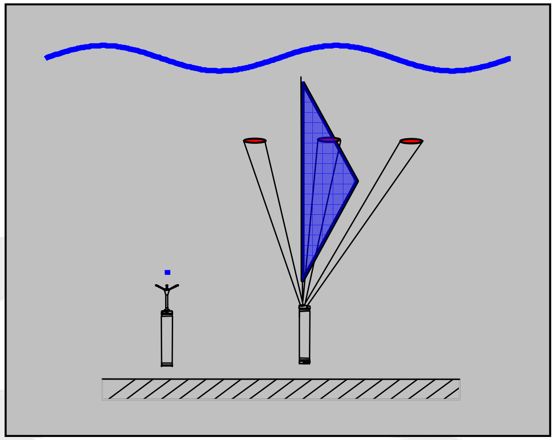
**Essentially removed the depth limit for nondirectional ($H_s$ $T_p$)**
**However directional "array" limit remains.**

## SUV (2005)

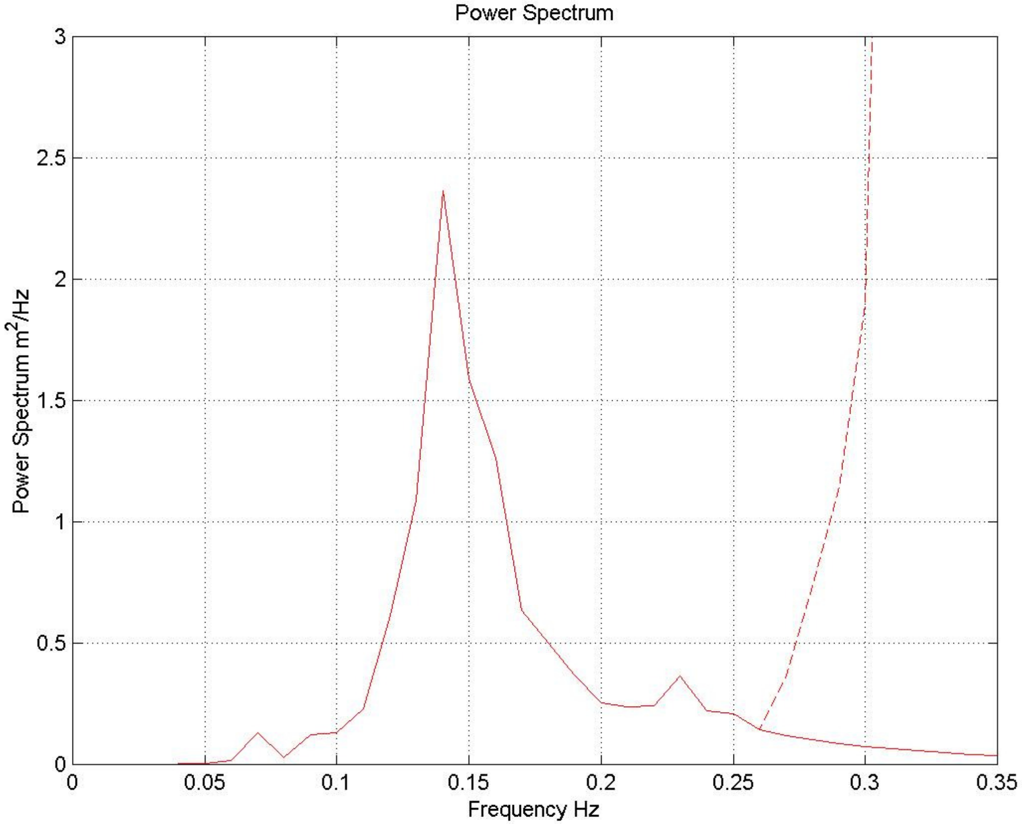
**Possibility for subsurface buoy mount**
**New directional solution based on direct measurements**

# PUV Method

- Aquadopp, Vector or Aquapro
- Measures Pressure and Horz Velocity (U and V)
- Out of Harms way
- Can collect data for months or days
- Straight forward Solution
- Applicable for Depths < 20 m
- Inferred solution



# Extrapolating Spectra

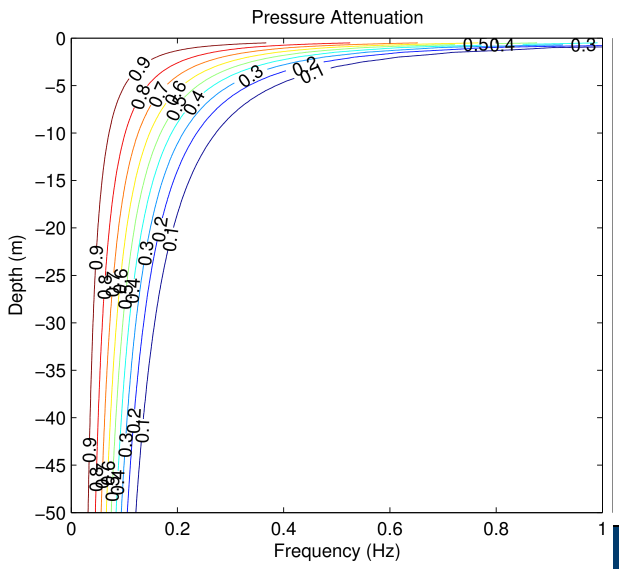



Apply Transfer Function to arrive at Power Spectra for Surface Elevation

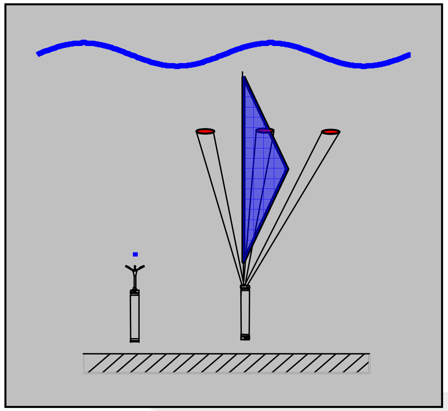
Quality control spectra via smoothing and extrapolating beyond cutoff freq

Use Power Spectra to Estimate Statistical Parameters (Hs, Tp)

$$S(f) = P(f)^2 / T(f)^2$$



# Estimated PUV Perfomance



| Depth | Period | Hs |
|---|---|---|
| 20 | 5.8 | 0.75 |
| 20 | 6.4 | 0.60 |
| 20 | 7.2 | 0.35 |
| 20 | 8.3 | 0.30 |
| 20 | 10.1 | 0.20 |
| 15 | 5.0 | 0.50 |
| 15 | 5.5 | 0.40 |
| 15 | 6.2 | 0.25 |
| 15 | 7.1 | 0.20 |
| 15 | 8.7 | 0.15 |
| 10 | 4.1 | 0.40 |
| 10 | 4.5 | 0.30 |
| 10 | 5.0 | 0.20 |
| 10 | 5.8 | 0.15 |
| 10 | 7.1 | 0.10 |
| 5 | 2.9 | 0.20 |
| 5 | 3.2 | 0.15 |
| 5 | 3.6 | 0.10 |
| 5 | 4.1 | 0.07 |
| 5 | 5.0 | 0.05 |
| 3 | 2.3 | 0.12 |
| 3 | 2.5 | 0.10 |
| 3 | 2.8 | 0.05 |
| 3 | 3.2 | 0.04 |
| 3 | 3.9 | 0.03 |

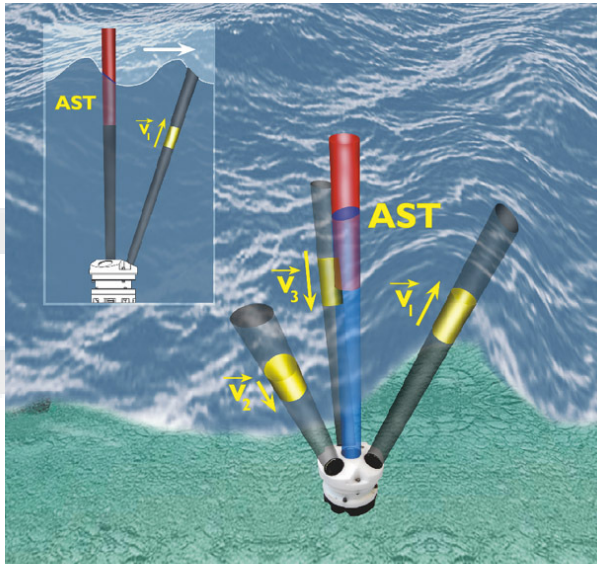
# AWAC: Waves and Current Profiles In one instrument

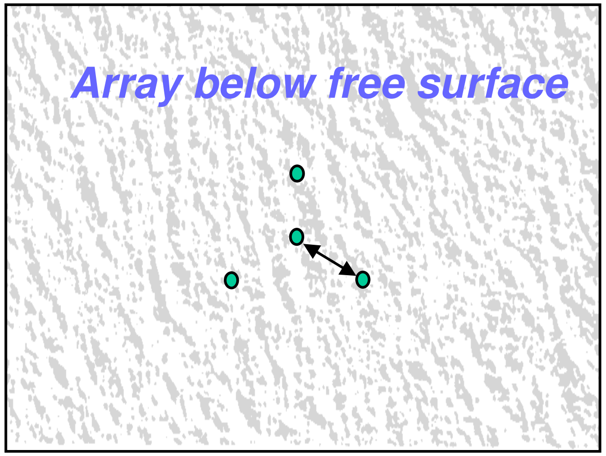
- Alternating Profile/Wave modes

- Projects and *"Array"* just below the surface

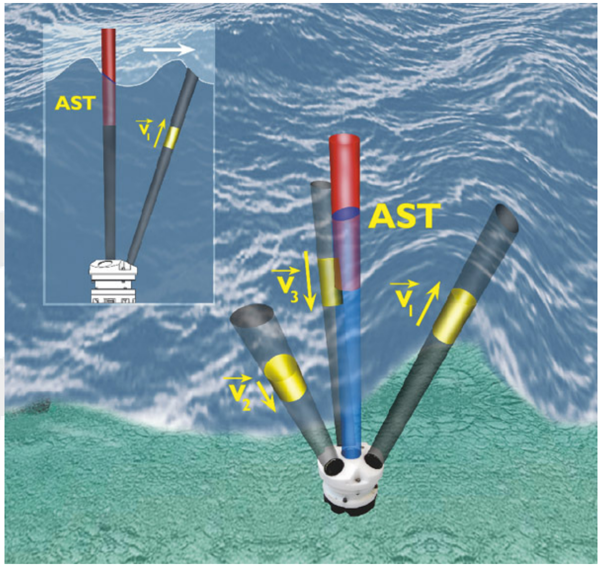
- Special wave processing: Maximum Likelihood Method

- Nondirectional solution is coupled to the Directional solution

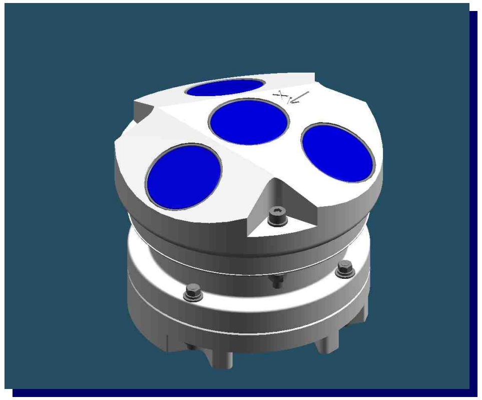


# AWAC: Waves and Current Profiles In one instrument




- 1 MHz or 600 kHz Transmit
- Measure current profile 25/50 m range
- Wave measurements 35/60 m range
- Sample at 2 Hz (or 4 Hz 1 MHz only)
- Self-contained w/ internal recorder & batteries
  1 burst/hour setup provides 6 weeks for 1 battery pack, 26 Mb
- Online data collection (cable, GSM, radio,
  Internal Processor, acoustic modem)

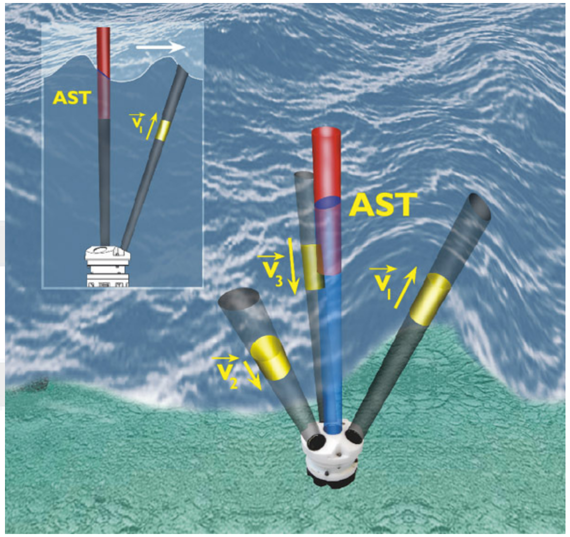


# Maximum Likelihood Method ... In short

- **MLM is a general method for solving directional wave spectra for a spatial array**

- **Use time-lags between array measurements to estimate directional distribution**

- **Transfer function for each velocity measurement is directionally dependent**

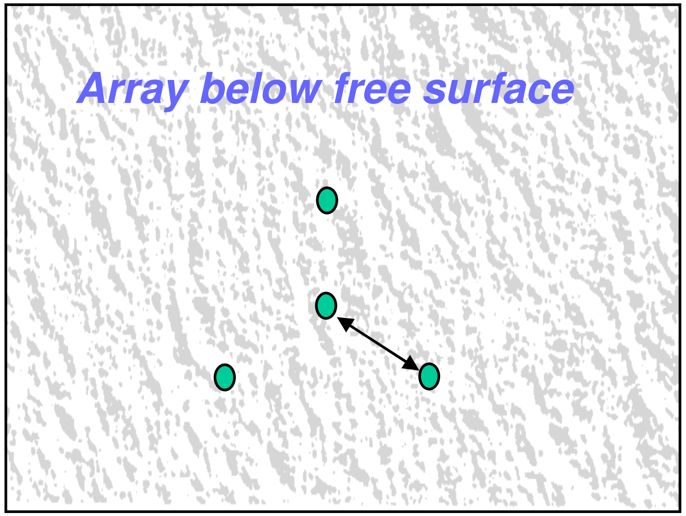



$$D(f,\theta) = \frac{\kappa}{H(f,\theta)^{*t}\,\Phi(f)^{-1}\,H(f,\theta)}$$

$H$ =Transfer function

$\Phi$ = Cross Spectra

$\kappa$ = Normalization constant

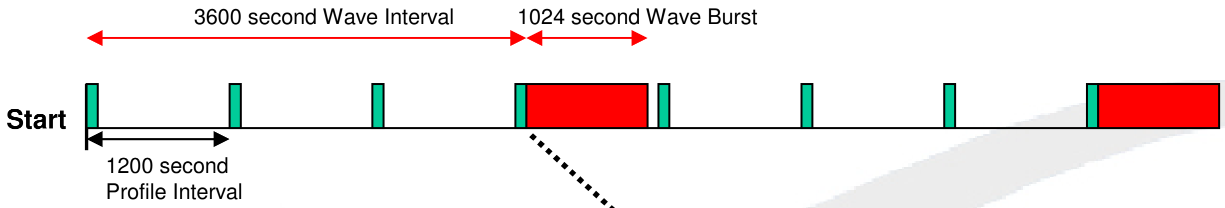


# Adaptive Measurement Volumes

## Measurement Intervals

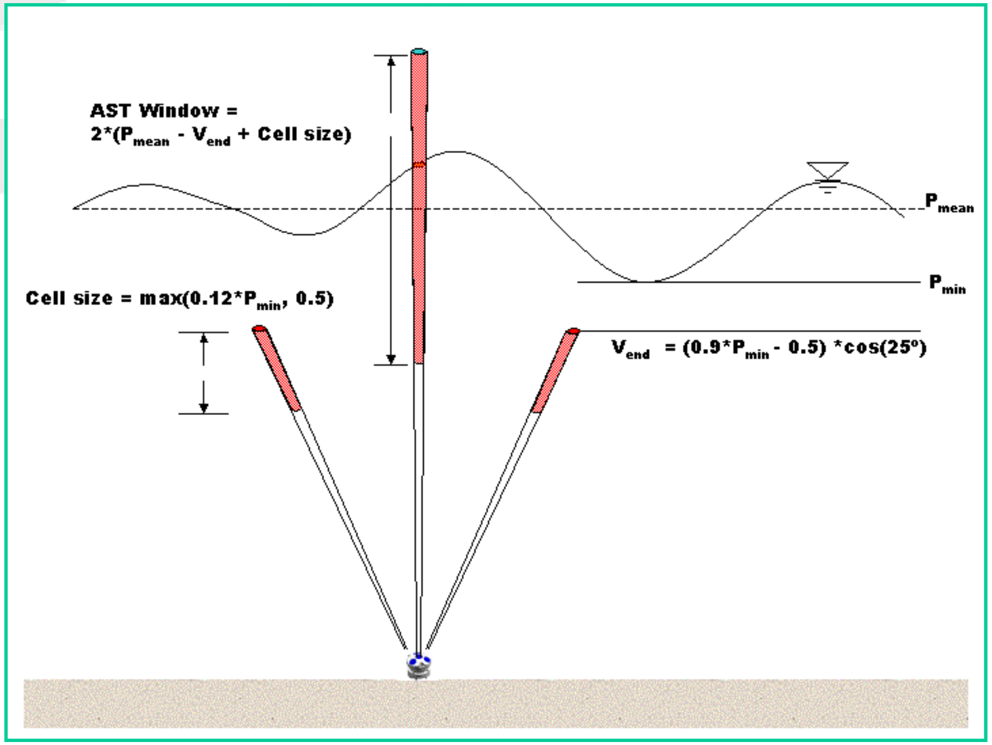
Profile Mode

Wave Mode




## Current Profile:

1. AST Window
2. Cell size
3. Cell position
4. SNR test



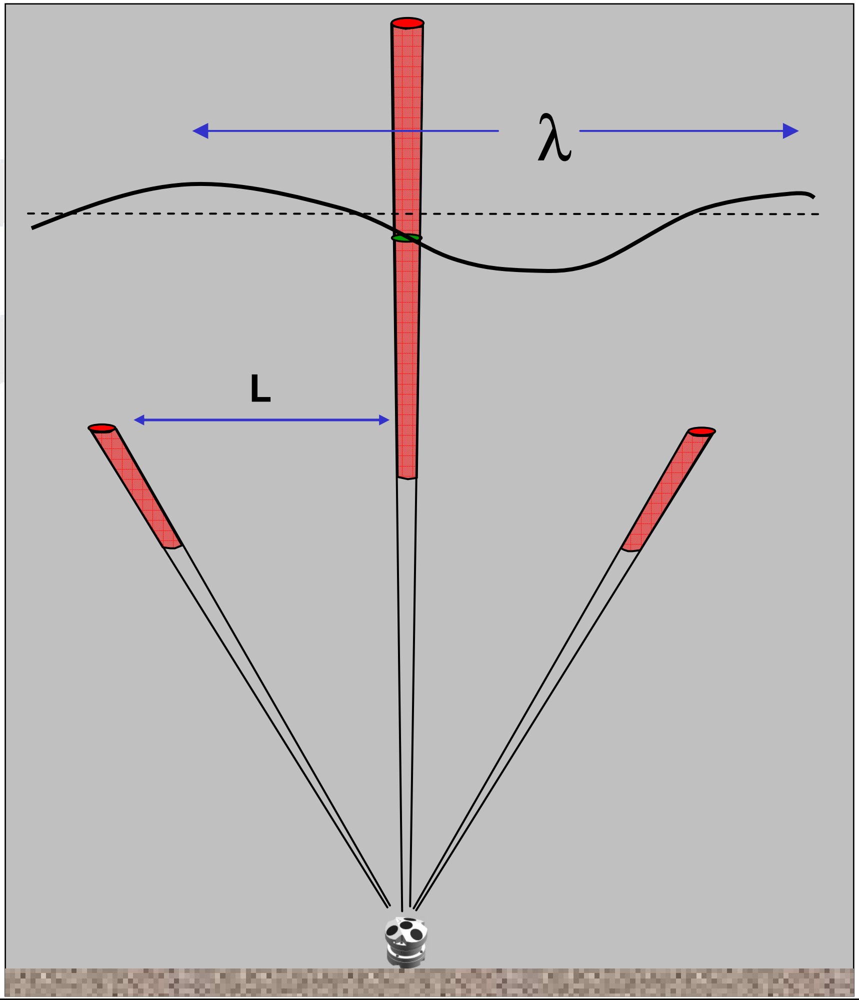
# Depth Limited Array

$L < \lambda/2$

**Geometric limitation**

**Measurement cell separation must be less then half wavelength**

**Separation increases for greater Depths**

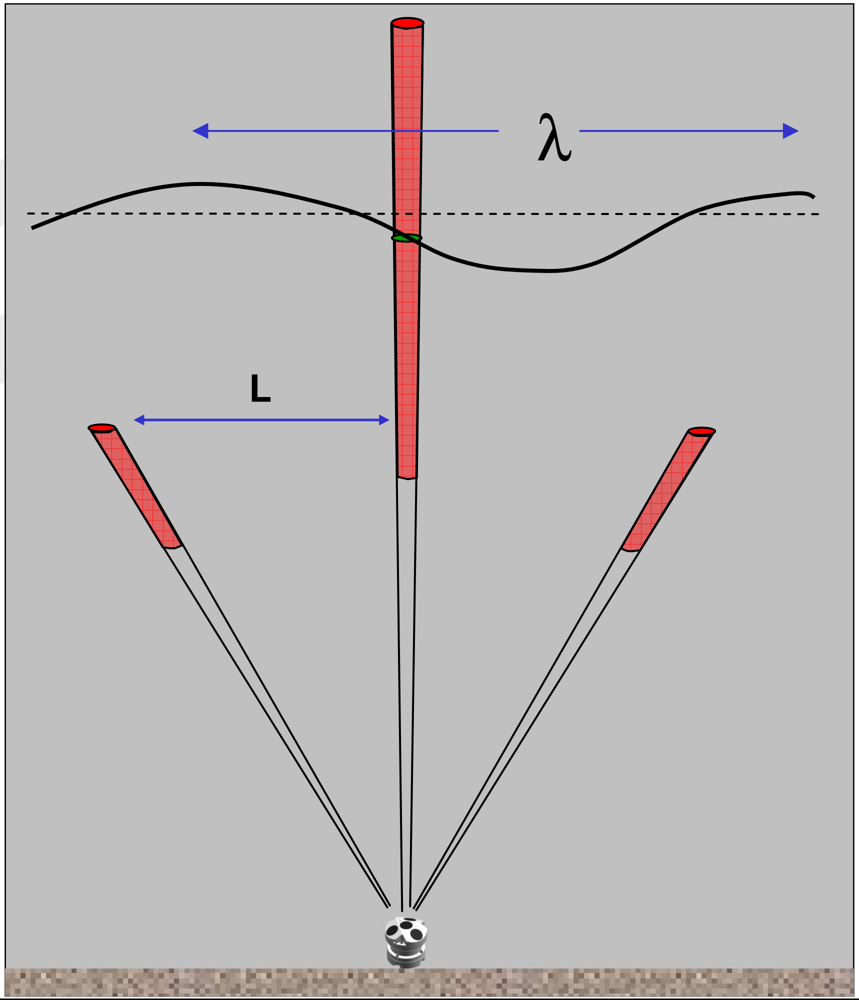


# Depth Limited Array

| Depth | Cutoff Freq | Cutoff Period |
|---|---|---|
| 5 | 0.7 | 1.45 |
| 10 | 0.45 | 2.20 |
| 20 | 0.32 | 3.10 |
| 30 | 0.26 | 3.85 |
| 40 | 0.23 | 4.35 |
| 50 | 0.20 | 5.0 |
| 60 | 0.18 | 5.55 |

- **Non-directional solution coupled to directional solution**

$L < \lambda/2$



# End of Intro